Granger-Causality Test in De-trended and Differenced Data

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トレンドと単位根を除去したグレンジャー因果性テスト

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Abstract A modified test for Granger-causality relationships between nonlinear time series variables with unit-root and time-trend components is proposed. It is because most macroeconomic time series are involved in a unit root and a time trend component. The modified test indicates no unidirectional causality from money to real GDP, which supports the quantitative theory of money. In contrast, conventional test via only differencing the time series often shows unidirectional causality relations from money to real GDP.

Keywords: Spurious Granger causality; Unit root; Time trend; Money supply; Output
キーワード：見かけ上のグレンジャーの因果関係；単位根；時間トレンド；マネーサプライ；産出

Introduction

The causality from money to output has been a long standing puzzle in econometrics. Standard F statistics for testing Granger causality requires that the time series investigated must be stationary. Unfortunately, most macroeconomic time series are involved in time trend and unit root (i.e. non-stationary) components (Nelson and Plosser, 1982). It has been demonstrated that standard F statistics for testing Granger causality often lead to spurious causality when we use non-stationary data (Granger and Newbold, 1974; Stock and Watson, 1989; Toda and Phillips, 1993), particularly one of the two series has a deterministic trend (He and Maekawa, 2001). Conventional causality test is performed after differencing the time series. However, whether differencing a time series simultaneously removes the time trend of the data remains unknown. For this reason, this paper proposes a modified test for Granger causality by both de-trending and differencing the time series.

This test is conducted to detect directional Granger-causality relationships between the US money supply and velocity and real GDP. Empirical result indicates no un-idirectional causality from money to real GDP, which supports economic theory. The quantitative theory of money posits that there should not be significant unidirectional causality from money to real product in the long-run.

Methods

The Granger-causality test is a statistical hypothesis test for determining whether one time series is useful in forecasting another. The statement that X_t does Granger-cause Y_t is understood that we are better able to predict Y_t using all available information of past X_t and past Y_t than only past Y_t (Granger, 1969). We consider the de-trended and differenced time series $\{\triangle y_t\}$ and $\{\triangle x_t\}$ instead of the time series $\{Y_t\}$ and $\{X_t\}$. Then, Granger-causality test is based on the regressions

$$\triangle y_t = \sum_{i=1}^{k} a_i \triangle y_{t-i} + \sum_{j=1}^{l} b_j \triangle x_{t-j} + u_{1t},$$
$$\triangle x_t = \sum_{j=1}^{l} c_j \triangle x_{t-j} + \sum_{i=1}^{k} d_i \triangle y_{t-i} + u_{2t},$$
$$t = 1, 2, \cdots, T$$

We can test the null hypothesis

$$H_0^{(1)} : b_1 = b_2 = \cdots = b_l = 0$$

(i.e. X_t does not Granger-cause Y_t) by the F statistic F_0 :

$$F_0 = \frac{(Q_3 - Q_1)/l}{(Q_5 - Q_3)/(n - k - l + 1)},$$

where Q_1 denotes the sum of squares of forecasted values of y obtained by regressing y on y_{-i}, Q_3 denotes the sum of squares of forecasted values of y obtained by regressing y on y_{-i} and x_{-j} , and Q_5 denotes the sum of squares of the sample values of y. We can denote

$$
\begin{aligned}
Q_1 &= y'Y(Y'Y)^{-1}Y'y,\\
Q_3 &= y'X(X'X)^{-1}X'y,\\
Q_5 &= y'y,\\
Y &= (y_{-1}, y_{-2}, \cdots, y_{-k}),\\
X &= (y_{-1}, y_{-2}, \cdots, y_{-k}, x_{-1}, x_{-2}, \cdots, x_{-l}),
\end{aligned}
$$

where

$$
\begin{aligned}
y &= (\triangle\ y_1, \triangle\ y_2, \cdots, \triangle\ y_T)',\ x = (\triangle\ x_1, \triangle\ x_2, \cdots, \triangle\ x_T)',\\
y_{-i} &= (\triangle\ y_{-(i-1)}, \triangle\ y_{-(i-2)}, \cdots, \triangle\ y_{T-i})',\ i = 1, 2, \cdots, k,\\
x_{-j} &= (\triangle\ x_{-(j-1)}, \triangle\ x_{-(j-2)}, \cdots, \triangle\ x_{T-j})',\ j = 1, 2, \cdots, l.
\end{aligned}
$$

If $H_0^{(1)}$ is rejected, we say that X_t does Granger-cause Y_t.

We can also test the null hypothesis

$$
H_0^{(2)} : d_1 = d_2 = \cdots = d_k = 0
$$

(i.e. Y_t does not Granger-cause X_t) by the F statistic F_0' :

$$
\tilde{F}_0 = \frac{(\tilde{Q}_3 - \tilde{Q}_1)/k}{(\tilde{Q}_5 - \tilde{Q}_3)/(T - k - l + 1)},
$$

where $\tilde{Q}_1$ means the sum of squares of forecasted values of x obtained by regressing x on x_{-j}, $\tilde{Q}_3$ means the sum of squares of forecasted values of x obtained by regressing x on x_{-j} and y_{-i}, $\tilde{Q}_5$ means the sum of squares of the sample values of x. We can write

$$
\begin{aligned}
\tilde{Q}_1 &= x'X(X'X)^{-1}X'x,\\
\tilde{Q}_3 &= x'Y(Y'Y)^{-1}Y'x,\\
\tilde{Q}_5 &= x'x,\\
X &= (x_{-1}, x_{-2}, \cdots, x_{-l}),\\
Y &= (x_{-1}, x_{-2}, \cdots, x_{-l}, y_{-1}, y_{-2}, \cdots, y_{-k}),
\end{aligned}
$$

If $H_0^{(2)}$ is rejected we say that Y_t does Granger-cause X_t.

Empirical study

To illustrate this test method, we investigated the U.S. quarterly money supply/velocity (1959Q1-2010Q2 but 1959Q1-2006Q4 for M3SL and MSIM2). All data were transformed to natural logs, denoted by $\{Y_t\}$ and $\{X_t\}$. The procedure of two types of Granger-causality tests are as follows:

1) Detrend Y_t and X_t by running the regressions $Y_t = p_0 + p_1 t + \varepsilon_{1t}$ and $X_t = q_0 + q_1 t + \varepsilon_{2t}$. Estimate the coefficients $\hat{p}_0$, $\hat{p}_1$, $\hat{q}_0$, $\hat{q}_1$ of these detrending equations by the *OLS*. Denote the detrended residuals by $y_t \approx Y_t - \hat{p}_0 - \hat{p}_1 t$ and $x_t \approx X_t - \hat{q}_0 - \hat{q}_1 t$. Let $k = l = 8$. The detrended-differenced series are given by $\triangle y_{t-i} = y_{t-i} - y_{t-i-1}$ and $\triangle x_{t-j} = x_{t-j} - x_{t-j-1}$ for $i,j = 1,2,\cdots,8$.
2) Modified Test: Calculate the statistic F_0 and the statistic $\tilde{F}_0$ based on $\{\triangle y_t\}$ and $\{\triangle x_t\}$ obtained in Step 1 are used.
3) Conventional Test: Repeat Step 2 but $\triangle y_{t-i} = Y_{t-i} - Y_{t-i-1}$ and $\triangle x_{t-j} = X_{t-j} - X_{t-j-1}$ for $i,j = 1,2,\cdots,8$.

The modified causality test result presented in Table 1. The null hypothesis that money did not Granger-cause real GDP was often rejected in the conventional tests but could not be rejected in the detrended-differenced tests at 1% and 5% level as concluded in Table2.

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Table 1

F statistics for Granger causality tests based on the differenced ARI and detrended-differenced NALRI

Null Hypothesis	F-Statistic based on differenced series	F-Statstic based on detrended-differenced series
M1SL dos not Granger Cause GDPC1	1.9347	0.8584
GDPC1 dos not Granger Cause M1SL	6.087	2.9666
M2SL dos not Granger Cause GDPC1	3.0787	1.1258
GDPC1 dos not Granger Cause M2SL	1.8578	1.6786
M3SL dos not Granger Cause GDDPC1	2.1973	0.6707
GDPC1 dos not Granger Cause M3SL	0.3778	0.3103
M2MSL dos not Granger Cause GDPC1	4.4711	2.3071
GDPC1 dos not Granger Cause M2MSL	3.6192	2.9365
MZMSL dos not Granger Cause GDPC1	3.4908	1.6104
GDPC1 dos not Granger Cause MZMSL	2.7446	2.0029
NOM1M2 dos not Granger Cause GDPC1	2.9467	1.142
GDPC1 dos not Granger Cause NOM1M2	2.1828	2.8067
MSIM2 dos not Granger Cause GDPC1	4.7411	2.1783
GDPC1 dos not Granger Cause MSIM2	1.7369	2.102
M1V dos not Granger Cause GDPC1	1.5794	1.083
GDPC1 dos not Granger Cause M1V	3.0333	2.6284
M2V dos not Granger Cause GDPC1	3.3064	2.0562
GDPC1 dos not Granger Cause M2V	2.4579	2.4716
MZMV dos not Granger Cause GDPC1	2.8582	1.8465
GDPC1 dos not Granger Cause MZMV	2.432	2.4394

Note: The sample size T=206 but T=188 for M3SL and MSIM2. F(8, 200)=1.98 at 5%. F(8, 200)=2.60 at 1%.

Table 2

Test results for unidirectional causations from money to product and bidirectional causations between money and product

Null hypothesis is rejected	
at the 1% significance level	at the 5% significance level
Based on the differenced ARI	
Unidirectional causation from M2SL to GDPC1	Unidirectional causation from M3SL to GDPC1
Bidirectional causation between M2MSL and GDPC1	Bidirectional causation between NOM1M2 and GDPC1
Bidirectional causation between MZMSL and GDPC1	Bidirectional causation between M2V and GDPC1
Unidirectional causation from NOM1M2 to GDPC1	Bidirectional causation between MZMV and GDPC1
Unidirectional causation from MSIM2 to GDPC1	
Unidirectional causation from M2V to GDPC1	
Unidirectional causation from MZMV to GDPC1	
Based on the detrended-differenced NLARI	
	Bidirectional causation between M2MSL and GDPC1
	Bidirectional causation between MSIM2 and GDPC1
	Bidirectional causation between M2V and GDPC1