

## TWO-MESON ANNIHILATION OF A NUCLEON AND ANTINUCLEON

E.M. HENLEY, T. OKA and J. VERGADOS<sup>1</sup>*Institute for Nuclear Theory, Department of Physics, FM-15, University of Washington, Seattle, WA 98195, USA*

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Low energy nucleon–antinucleon annihilation into two mesons is examined in a distorted wave, “effective” perturbative QCD, approach. Both velocity-independent and first-order velocity-dependent terms are kept in the basic interaction responsible for the quark–antiquark annihilation. Interesting selection rules are predicted. Cross sections are evaluated.

Studies of nucleon–antinucleon ( $N\bar{N}$ ) annihilation are of considerable interest. Even at low energies, this annihilation requires overlap of the two particles; it is thus likely that the process is sensitive to quark dynamics and a study of it may lead to a better understanding of the role of quarks in nuclear physics. Early work [1] showed that the cross sections for annihilation, elastic scattering, and charge exchange are dependent primarily on geometrical considerations (e.g. size of quark confinement region). However, the rates for annihilation into specific channels are more likely to be dependent on the reaction dynamics and on quark confinement properties. In this letter we examine the two-meson annihilation channels because these processes can be and are now being studied in some detail at LEAR [2].

We report on an investigation of a distorted wave approach. The distortion is provided by a long range meson-exchange potential with absorption, and the perturbing interaction is an effective QCD one-gluon mechanism which provides the  $N\bar{N}$  annihilation into two mesons. Our study is part of a larger one which is to be contrasted to the more “standard”  $^3P_0$  model in which a quark–antiquark ( $q\bar{q}$ ) is created (annihilated) from (into) the vacuum [3]. In our model the basic dynamics is a one-gluon-exchange mechanism as shown in fig. 1. It is a distorted wave “effective” QCD perturbative model in that we assume an aver-

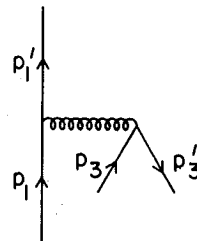


Fig. 1. The diagram which gives rise to the annihilation of a  $q\bar{q}$  in the one-gluon-exchange approximation.

age strong-coupling constant,  $\alpha_s$ , rather than a running one as required by basic QCD. A similar approach has been used by Kohno and Weise [4], and Maruyama and Ueda [5], but our work differs from theirs in philosophy and intent, as well as in details. In particular, we focus on seeing how well the effective QCD approach corresponds to nature, whereas they include other reaction mechanisms and approximations, so that the nature of the dominant mechanism responsible for the annihilation is lost. Our approach is similar to the successful one-boson-exchange models for the nucleon–nucleon force and to distorted wave treatments of direct nuclear reactions. An earlier report [6] focused on vector meson decays, and we have also examined meson–nucleon vertex functions in our model [7].

The reader may well wonder why an effective one-gluon-exchange mechanism should be applicable to  $N\bar{N}$  annihilation. For instance, strange-meson (e.g.  $K\bar{K}$ ) production requires at least two gluons and is of

<sup>1</sup> Permanent and present address: Physics Department, University of Ioannina, GR 45332 Ioannina, Greece.

order  $\alpha_s^2$ . We adopt the premise that the one-gluon (order  $\alpha_s$ ) processes are the dominant ones and wish to examine to what extent our model predicts the main features of experimental results. That is, we assume that order  $\alpha_s^2$  processes are small corrections (<25%) to our model. Our philosophy is that the many-gluon effects are responsible for confinement and give constituent quarks their effective masses, but do not contribute to  $q\bar{q}$  annihilation or production.

The basic diagram of our model is that of fig. 1. We assume constituent quarks, and for simplicity use a nonrelativistic reduction of the matrix element associated with fig. 1; that is, we keep only terms of order  $m^{-1}$ , where  $m$  is the constituent quark mass. Relativistic effects can be incorporated, but we believe that the main features of our model are contained in the nonrelativistic approximation. In this case, we obtain in momentum space

$$v_{13}(q, p_1) = -g_s^2 \frac{1}{4} \lambda_1 \cdot \lambda_3 (\omega_q^2 - q^2)^{-1} \times \{ \frac{1}{2} (1/m_3 - 1/m_1) \sigma_3 \cdot q - (\sigma_3 \cdot p_1)/m_1 + [i(\sigma_1 \times \sigma_3)/2m_1] \cdot q \}, \quad (1a)$$

where  $q = p'_1 - p_1 = p_3 + p'_3$ ,  $\omega_q = E_{p'_1} - E_{p_1}$ ,  $\sigma_i$  is the spin of quark  $i$  (see fig. 1),  $m_i$  is its mass,  $g_s$  is the strong-coupling constant and  $\frac{1}{4} \lambda_1 \cdot \lambda_3$  is the color factor. The underlined 3 indicates the quark which is annihilated.

We evaluate the  $N\bar{N}$  annihilation matrix element for the case shown in fig. 2 and for similar diagrams in which the gluon connects to other quarks or antiquarks. We obtain

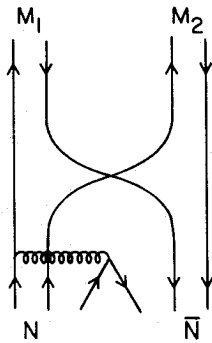


Fig. 2. The diagram for  $N\bar{N}$  annihilation into two mesons in the one-gluon-exchange model.

$$\mathcal{M} = -3 \langle M_1(1\bar{2}) M_2(2\bar{1}) | \sum_{i=1,2,\bar{1},\bar{2}} v_{i3} | N(123) \bar{N}(\bar{1}\bar{2}\bar{3}) \rangle, \quad (2)$$

where the factor of  $-3$  arises from the sum over  $q\bar{q}$  pairs (e.g.  $3\bar{3}$ ,  $3\bar{2}$ ,  $3\bar{1}$ , ...) which can annihilate.

We assume that the quarks in the nucleon are solely in spatial S-states [8]. The  $\omega$  meson is known to be almost a pure  $u$  and  $d$  quark state, since the  $\phi$  meson is an almost pure  $s\bar{s}$ , where  $s$  denotes a strange quark [8]. The  $\eta$  and  $\eta'$ , on the other hand, are almost pure octet and singlet, respectively, so that the isospin zero combination is

$$\bar{\eta} \equiv (1/\sqrt{2})(u\bar{u} + d\bar{d}) = (1/\sqrt{3})\eta + \sqrt{\frac{2}{3}}\eta'. \quad (3)$$

It is this combination,  $\bar{\eta}$ , to which decays will be computed.

We make use of the symmetry of the wavefunctions to evaluate the matrix element  $\mathcal{M}$ , which factors into

$$\mathcal{M} = \mathcal{M}_{\text{space}} \mathcal{M}_{\text{SI}} \mathcal{M}_C, \quad (4)$$

where the subscript SI stands for spin-isospin.

For the color matrix element  $\frac{1}{4} \lambda_1 \cdot \lambda_3$  we obtain

$$\mathcal{M}_C(\text{particle}) = -\frac{2}{9}, \quad \mathcal{M}_C(\text{antiparticle}) = \frac{2}{9}, \quad (5a, b)$$

where (5a) and (5b) refer, respectively, to gluon couplings to quarks  $i=1,2$  and antiquarks  $\bar{1},\bar{2}$  in eq. (2).

The spin-isospin parts of the matrix element are somewhat more complicated to evaluate, but it can be done in a straightforward manner by means of angular momentum coupling and recoupling coefficients. Details will be presented in a longer communication. Here we simply quote the final result.

For our purpose, it is useful to rewrite eq. (1) so as to separate out the spin dependence. We write

$$v_{13} = \frac{1}{4} \lambda_1 \cdot \lambda_3 (\bar{\sigma}_3 \cdot A_{13} + i \sigma_1 \times \sigma_3 \cdot B_{13}). \quad (1b)$$

Since we neglect up-down quark mass differences ( $m_1 = m_3 = m$ ), we have

$$A_{13} = g_s^2 (\omega_q^2 - q^2)^{-1} p_1/m, \quad (6a)$$

and

$$B_{13} = -g_s^2 (\omega_q^2 - q^2)^{-1} q/2m. \quad (6b)$$

We thus obtain for the matrix  $\mathcal{M}$  of eq. (2)

$$\mathcal{M} = (-1)^{N-M} (S'1MN - M1SN) \times (\mathcal{M}_{\text{SIC}}^A A^{M-N} + \mathcal{M}_{\text{SIC}}^B B^{M-N}), \quad (7)$$

where  $S'$  ( $S$ ) is the initial (final) state spin and  $M(N)$  is the corresponding magnetic quantum number. The matrices  $\mathcal{M}_{\text{SIC}}^A$  and  $\mathcal{M}_{\text{SIC}}^B$  are

$$\begin{aligned} \mathcal{M}_{\text{SIC}}^A = & 16\sqrt{3}(-1)^{S'-S}[1+(-1)^L](\bar{I}_2\bar{I}_2\bar{I}_1\bar{S}_2\bar{S}')^{1/2} \\ & \times \sum_{l,l'} \bar{l}\bar{l}' \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & I_1 \\ \frac{1}{2} & \frac{1}{2} & I_2 \\ l & l' & I \end{pmatrix} \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & S_1 \\ \frac{1}{2} & \frac{1}{2} & S_2 \\ l & l' & S \end{pmatrix} \\ & \times \begin{pmatrix} l & \frac{1}{2} & \frac{1}{2} \\ l' & \frac{1}{2} & \frac{1}{2} \\ I & 0 & I \end{pmatrix} \begin{pmatrix} l & \frac{1}{2} & \frac{1}{2} \\ l' & \frac{1}{2} & \frac{1}{2} \\ S & 1 & S' \end{pmatrix}, \end{aligned} \quad (8a)$$

and

$$\begin{aligned} \mathcal{M}_{\text{SIC}}^B = & 96\sqrt{3}(-1)^S(\bar{I}_1\bar{I}_2\bar{I}_1\bar{S}_2\bar{S}')^{1/2} \\ & \times \sum_{l'k'} (-1)^{l'} \sum_{lk} [(-1)^L + (-1)^{l+k}] \\ & \times \bar{l}\bar{l}'\bar{k}\bar{k}' \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & I_1 \\ \frac{1}{2} & \frac{1}{2} & I_2 \\ l & l' & I \end{pmatrix} \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & S_1 \\ \frac{1}{2} & \frac{1}{2} & S_2 \\ k & l' & S \end{pmatrix} \\ & \times \begin{pmatrix} l & \frac{1}{2} & \frac{1}{2} \\ l' & \frac{1}{2} & \frac{1}{2} \\ I & 0 & I \end{pmatrix} \begin{pmatrix} l & \frac{1}{2} & \frac{1}{2} \\ l' & \frac{1}{2} & \frac{1}{2} \\ k' & 1 & S' \end{pmatrix} \\ & \times \begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & k & l \end{pmatrix} \begin{pmatrix} 1 & l & k \\ l' & S & k' \end{pmatrix} \begin{pmatrix} S' & k' & 1 \\ 1 & 1 & S \end{pmatrix}, \end{aligned} \quad (8b)$$

where the  $\{ \}$  are  $6j$ - or  $9j$ -coefficients,  $\bar{K} = 2K + 1$  ( $K = I_1, I_2, \dots, k, k'$ ) and  $L$  is the relative orbital angular momentum of the two mesons  $M_1$  and  $M_2$ . We have used  $G$  parity conservation in arriving at eqs. (8). From them follow striking selection rules which differ from the  ${}^3P_0$  model and which are shown in tables 1 and 2 for annihilation from S- and P-states of the  $\text{NN}$  system. The momentum-dependent matrix element does not contribute to two-meson annihilation from  $\text{NN}$ -S-states, but only from P-states. This momentum-dependent contribution to the two-meson annihilation of the  $\text{NN}$  system was omitted by Kohno and Weise [4] and by Maruyama and Ueda [5]. On the other hand, the static matrix element considered by these authors vanishes for annihilation from the  ${}^3P_0$

Table 1

The (reduced in spin) matrix elements  $\mathcal{M}_{\text{SIC}}^A$  and  $\mathcal{M}_{\text{SIC}}^B$  appropriate for S-state  $\text{NN}$  annihilation.

Initial NN states	$I$	Final states		$\mathcal{M}_{\text{SIC}}^A$	$\mathcal{M}_{\text{SIC}}^B$
		$S$	mesons		
$S' = 0$	0	1	$\rho\rho$	0	$+4\sqrt{6}/27$
	0	1	$\omega\omega$	0	$+4\sqrt{2}/9$
	1	1	$\pi^\pm\rho^\mp$	0	$-8\sqrt{2}/9$
	1	1	$\rho^0\omega$	0	$+16\sqrt{2}/27$
$S' = 1$	0	1	$\pi\rho$	0	$+44\sqrt{6}/27$
	0	1	$\tilde{\eta}\omega$	0	$-4\sqrt{2}/9$
	1	0	$\pi^\pm\pi^\mp$	0	$+8\sqrt{6}/9$
	1	0	$\rho^\pm\rho^\mp$	0	$+8\sqrt{2}/9$
	1	1	$\pi^0\omega$	0	$+28\sqrt{2}/27$
	1	1	$\tilde{\eta}\rho^0$	0	$+4\sqrt{2}/27$
	1	2	$\rho^\pm\rho^\mp$	0	$-4\sqrt{2}/9$

and  ${}^3P_2$  states to a final spin 0 state of two mesons. The selection rules can distinguish between our model, and that of Kohno and Weise or Maruyama and Ueda, and the  ${}^3P_0$  model. In the latter case, to order  $\alpha_s$  there is no two-meson annihilation from S-states of the  $\text{NN}$  system.

Table 2

The (reduced in spin) matrix elements  $\mathcal{M}_{\text{SIC}}^A$  and  $\mathcal{M}_{\text{SIC}}^B$  relevant for P-state  $\text{NN}$  annihilation.

Initial NN states	$I$	Final states		$\mathcal{M}_{\text{SIC}}^A$	$\mathcal{M}_{\text{SIC}}^B$
		$S$	mesons		
$S' = 0$	0	1	$\pi\rho$	$+8\sqrt{3}/27$	$-8\sqrt{3}/27$
	0	1	$\tilde{\eta}\omega$	$+8/9$	$-8/9$
	1	1	$\pi^0\omega$	$+40/27$	$-40/27$
	1	1	$\tilde{\eta}\rho^0$	$-8/27$	$+8/27$
	1	1	$\rho^\pm\rho^\mp$	$-8/9$	$+8/9$
$S' = 1$	0	0	$\pi\pi$	$-20/9$	0
	0	0	$\tilde{\eta}\tilde{\eta}$	$+4\sqrt{3}/9$	0
	0	0	$\rho\rho$	$-52\sqrt{3}/27$	0
	0	0	$\omega\omega$	$+20/9$	0
	1	0	$\pi^0\tilde{\eta}$	$-8\sqrt{3}/27$	0
	1	0	$\rho^0\omega$	$+8/27$	0
	1	1	$\pi^\pm\rho^\mp$	$-16/9$	$+8/9$
	1	1	$\rho^0\omega$	$+16/9$	$-8/9$
	0	2	$\rho\rho$	$+8\sqrt{3}/27$	$-4\sqrt{3}/9$
	0	2	$\omega\omega$	$+8/9$	$-4/3$
	1	2	$\rho^0\omega$	$+32/27$	$-16/9$

Table 3

Cross sections in mb for  $N\bar{N}$  annihilation at  $P_{\text{Lab}} = 100$  MeV/c to two mesons. Columns 2–4 are for  $\alpha_s = 2$  and  $R_N = 0.6$  fm, columns 2 and 3 are for equal harmonic oscillator strengths for the nucleon and mesons; columns 4 and 5 are for a smaller meson radius. Columns 2 and 4 are for a static interaction only, whereas 3 and 5 are for the full interaction, eq. (1). Columns 6 and 7 correspond to ref. [4] and ref. [5], respectively, normalized to a  $\pi^+\pi^-$  cross section of unity.

Mesons	$R_m = \sqrt{3}R_N/2 = 0.52$ fm		$R_m = 0.4$ fm		Ref. [4]	Ref. [5]
	static	with momentum dependence	static	with momentum dependence		
$\pi^+\pi^-$	6.9	7.1	4.1	5.4	1	1
$\pi^0\pi^0$	0	0.08	0	0.64	0	0
$\pi^0\eta$	0	0.02	0	0.02	0	0
$\eta\eta$	0	0.02	0	0.15	0	0
$\pi^+\rho^-$	33	39	28	34	11.5	12
$\pi^0\rho^0$	26	27	24	24	7.1	10
$\pi^0\omega$	21	23	12	15	4.3	5
$\eta\rho^0$	0.30	0.41	0.19	0.27		0.2
$\eta\omega$	2.0	4.0	1.9	3.3		2
$\rho^+\rho^-$	62	93	41	163	19.6	18
$\rho^0\rho^0$	5.0	12	4.6	60	7.1	1
$\rho^0\omega$	39	146	33	129	17.6	6
$\omega\omega$	48	65	45	125	34.9	15

In order to obtain cross sections for the various two-meson channels listed in tables 1 and 2, we use distorted waves in the initial  $N\bar{N}$  states at a laboratory momentum of 100 MeV/c. This momentum is chosen so as to be able to compare our results with those of ref. [4]. The distorted waves are obtained from the work of Alberget al. [1]. Final state distortion effects are neglected. For the confined gluon propagator, which carries both energy ( $q_0$  and three-momentum ( $\mathbf{q}$ ), we neglect the three-momentum transfer and assume  $\omega_q \approx 2m$  because  $q_0$  is always larger than  $|\mathbf{q}|$ .

Although the non-vanishing spin–isospin matrix elements of tables 1 and 2 have large variations, they still must be multiplied by spatial matrices and by phase space factors. These two factors, particularly the former, greatly influence the calculated exclusive two-meson production cross section, as shown in table 3. In this table we list separately the results obtained for the contribution to the cross sections from the static term alone (columns 2 and 4) and from the sum of the static and momentum-dependent interaction terms (columns 3 and 5). This separation is carried out so that we can compare our results to those of ref. [4] and ref. [5]. The momentum-dependent term gives rise to small cross sections for neutral pseudo-

scalar mesons, where the static term gives no contribution. The enhancement of cross sections from the momentum-dependent term varies with the masses of the mesons produced, and is largest for the heaviest (vector) mesons. The value of  $\alpha_s$  is taken to be 2 to give reasonable cross sections, in rough agreement with our earlier work [6] and ref. [4]. Our results can be compared to those of ref. [4] and ref. [5] (columns 6 and 7) by normalizing our  $\pi^+\pi^-$  cross section to unity. In ref. [5] the cross sections are for annihilation at rest. An exact comparison is not possible because of the dependence on the  $N\bar{N}$  distorting potential and the hadronic radii. In columns 2 and 3 we have, for simplicity, chosen the harmonic oscillator strength parameters to be identical for all hadrons. Our columns 4 and 5 show the sensitivity to changes in the meson radius by reducing its value to 0.4 fm.

In our model, the annihilation into strange mesons is assumed to be sufficiently small that it can be calculated as a correction to the basic model. Recent data suggests that the ratio of the decay of the  $N\bar{N}$  to  $K^+K^-$  is about 15% of that to  $\pi^+\pi^-$  [4], which is not in disagreement with our model, particularly since the cross section for annihilation into  $\pi^+\pi^-$  already is very small. The suppression of this annihilation rate is

sensitive to the radius of the quark confinement region in the nucleon. On the other hand, earlier bubble chamber data [9] indicated a larger  $K^+K^-/\pi^+\pi^-$  ratio, which would be troublesome for our model. Also recent experiments at LEAR indicate a large cross section for the decay to the  $\rho\pi$  channel [10], which is in accord with our model. Large cross sections are also predicted for the production of  $\pi\omega$  and pairs of vector mesons. It will be interesting to see whether these predictions of our model, which differ from those of the  $^3P_0$  model, are obtained experimentally. The lack of annihilation from S-states into two mesons to first order in the interaction in the latter model is one of the distinct differences. This annihilation only occurs to third order in the interaction and with an adjustable strength.

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